Day 20
Mon Jun 19
Is it possible to write the vector (-5,6) as a linear combination of (1,2) and (3,4)?

We want
\[ (-5, 6) = c_1 (1, 2) + c_2 (3, 4) \]

or
\[ -5 = 1 \cdot c_1 + 3 \cdot c_2 \]
\[ 6 = 2 \cdot c_1 + 4 \cdot c_2 \]

In matrix form:
\[
\begin{pmatrix}
1 & 3 \\
2 & 4
\end{pmatrix}
\begin{pmatrix}
c_1 \\
c_2
\end{pmatrix}
= \begin{pmatrix}
-5 \\
6
\end{pmatrix}
\]

\[
\det \begin{pmatrix}
1 & 3 \\
2 & 4
\end{pmatrix} = 4 - 6 = -2 \neq 0,
\]

so answer is Yes.

A unit vector is a vector of length 1.

The standard unit vectors are:
\[
i = (1, 0) \quad \hat{j} = (0, 1)
\]
$$\| i \| = \sqrt{1^2 + 0^2} = 1 \quad \| j \| = \sqrt{0^2 + 1^2} = 1$$

Fact: Any vector can be written as a linear combination of i and j in an easy way:

$$(4, 5) = 4(1, 0) + 5(0, 1)$$
$$= 4i + 5j$$

In general:

$$\left(x_1, x_2 \right) = x_1 i + x_2 j$$

Two vectors are said to be orthogonal when the angle between them is 90 degrees (so they are perpendicular).

From last time, the angle between u and v is given by:

$$\cos \Theta = \frac{u \cdot v}{\|u\| \|v\|}$$

So when u and v are orthogonal, then the dot product $u \cdot v = 0$ because $\cos(90^\circ) = 0$
In particular,

\[ \mathbf{i} \cdot \mathbf{j} = (1,0) \cdot (0,1) = 0 + 0 = 0 \]

so \( \mathbf{i} \) and \( \mathbf{j} \) are orthogonal.

Section 4.2
Visualizing vectors in \( \mathbb{R}^3 \)

A vector in 3 or more dimension can also be thought as an arrow, with the tail at the origin.

Ex: Plot the vector \((3,5,7)\).
Begin by plotting the three axes \(x\), \(y\) and \(z\) as follows:
Now draw the base of the box, then the sides, then draw the arrow.

\[(3, 5, 7) \times \gamma \hat{z}\]

The formulas we developed for vectors in 2 dimension carry over to 3 or higher dimensions. So we can now find the angle of vectors in 3 dimensions.
Example: find the angle between the diagonal of the previous box and the edge lying on the y-axis.

\[ \mathbf{v} = (3, 5, 7) \quad \mathbf{u} = (0, 5, 0) \]

\[ \mathbf{v} \cdot \mathbf{u} = 0 + 25 + 0 = 25 \]

\[ \|\mathbf{v}\| = \sqrt{3^2 + 5^2 + 7^2} = \sqrt{83} \]

\[ \|\mathbf{u}\| = \sqrt{5^2} = 5 \]

\[ \cos \theta = \frac{25}{\sqrt{83}} = \frac{5}{\sqrt{83}} \approx 0.5488 \]

\[ \theta = \arccos \frac{5}{\sqrt{83}} \approx 56.71^\circ \]

Cauchy-Schwartz inequality.

For any vectors \( \mathbf{u} \) and \( \mathbf{v} \) (in any dimension) we have the inequality:

\[ |\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\| \|\mathbf{v}\| \]

We can check the C-S inequality:

\[ \mathbf{v} = (2, -1, 5) \quad \mathbf{u} = (1, 3, -4) \]
\[ \mathbf{v} \cdot \mathbf{u} = 2 - 3 - 20 = -21 \]

\[ \| \mathbf{v} \| = \sqrt{2^2 + (-1)^2 + 5^2} = \sqrt{4 + 1 + 25} = \sqrt{30} \]

\[ \| \mathbf{w} \| = \sqrt{1^2 + 3^2 + (-4)^2} = \sqrt{26} \]

According to C-S:

\[ | -21 | \leq \sqrt{30} \cdot \sqrt{26} \quad \text{true} \]

HW p.244 # 7,21

turn in factoring problem on Wed.