Day 8
Mon May 22

from next time (Wed) we meet in room 306
Multiplication of matrices:

If $A$ is a square matrix, we can take powers:

$$A^{n \times n} \cdot B^{m \times p}$$

is defined, and it's $n \times p$

But powers are never defined for rectangular

So

$$B^{4 \times 5} \cdot (B^T)^{5 \times 4}$$

is defined, it's $3 \times 4$

If $A$ is a square matrix, we can take powers:

$$A^{n \times n} \cdot A^2 = A \cdot A^{n \times n}$$

is $n \times n$

$$A^3 = A^2 \cdot A$$

is $n \times n$

But powers are never defined for rectangular
matrices: \[
\begin{pmatrix}
A \\
3 \times 4
\end{pmatrix}
\quad A^2 = A A
\quad \text{not defined}
\]

But \( A A^T \) and \( A^T A \) are always defined:

\[
\begin{pmatrix}
A \\
\_\_\_\_\_\_ n \times m
\end{pmatrix}
\quad A^T
\quad \begin{pmatrix}
A^T \\
\_\_\_\_\_\_ m \times n
\end{pmatrix}
\quad A^T A
\quad \begin{pmatrix}
\_\_\_\_\_\_ m \times n
\end{pmatrix}
\quad \begin{pmatrix}
\_\_\_\_\_\_ n \times m
\end{pmatrix}
\]

In general multiplication of matrices is not commutative. \( A B \neq B A \)

But addition is commutative: \( A + B = B + A \)

A diagonal matrix is one like:

\[
\begin{pmatrix}
3 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3
\end{pmatrix}
\]

all entries not on main diagonal are zero.
Note: diagonal matrices are always square.
A diagonal matrix with no 0 on the diagonal is always in AREF

If there are 0's on diagonal, it may not be in AREF.